Applying Matrix Factorization to Consistency-based Direct Diagnosis

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Abstract Configuration systems must be able to deal with inconsistencies which can occur in different contexts. Especially in interactive settings, where users specify requirements and a constraint solver has to identify solutions, inconsistencies may more often arise. In inconsistency situations, there is a need of diagnosis methods that support the identification of minimal sets of constraints that have to be adapted or deleted in order to restore consistency. A diagnosis algorithm's performance can be evaluated in terms of time to find a diagnosis (runtime) and diagnosis quality. Runtime efficiency of diagnosis is especially crucial in real-time scenarios such as production scheduling, robot control, and communication networks. However, there is a trade off between diagnosis quality and the runtime efficiency of diagnostic reasoning. In this article, we deal with solving *the quality-runtime performance trade off problem* of direct diagnosis. In this context, we propose *a novel learning approach based on matrix factorization for constraint ordering*. We show that our approach improves runtime performance and diagnosis quality at the same time.

Keywords constraint satisfaction, diagnosis, matrix factorization, configuration systems

1 Introduction

Configuration systems [14] are used to find solutions for problems which have many variables and constraints. A configuration problem can be defined as a constraint satisfaction problem (*CSP*) [20]. If constraints of a *CSP* are inconsistent, no solution can be found. In this context, diagnosis is required to find at least one solution for an inconsistent *CSP* [1].

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There are several diagnosis approaches [13]. One approach is *direct diagnosis* which employs queries to check the consistency of a constraint set without the need to identify the corresponding conflict sets [5]. When diagnoses have to be provided in real-time, response times should be less than a few seconds [2]. For example, in communication networks, efficient diagnosis is crucial to retain the quality of service. However, there exists a trade-off between runtime performance of diagnosis calculation and diagnosis quality [4].

To address this challenge, we propose *Learned Constraint Ordering (LCO)* for direct diagnosis. Our approach learns constraint ordering heuristics from inconsistent historical transactions which include inconsistent user requirements. For example, our method learns that ”display quality” is more important than ”price” according to the past purchases and user requirements regarding a set of digital cameras. For this reason, ”display quality” is regarded as more important and ”price” would be a candidate for a change proposal. For example, a diagnosis would recommend to increase the price in order to be able to find an item that satisfies all requirements. Using historical inconsistent transactions, we build a sparse matrix and then employ matrix factorization techniques to estimate future diagnoses. After an offline learning phase, the most similar transaction to the new inconsistent requirement set is found and the corresponding constraint ordering heuristic (which has been calculated in the offline phase) is applied to reorder the inconsistent constraints before direct diagnosis is activated. Thanks to the learned ordering, direct diagnosis algorithms can solve a diagnosis task with a high prediction quality and shorter runtime compared to direct diagnosis without learned constraint ordering. We provide a working example to demonstrate the effects of our approach. Finally, based on experimental evaluations, we show that our constraint ordering approach with direct diagnosis is superior to the baseline (direct diagnosis algorithms without constraint ordering) on popular benchmark constraint satisfaction problems.

2 Preliminaries

In this section, we give an overview of the basic definitions in consistency-based configuration and diagnosis, and introduce a running example. Finally, we explain our evaluation criteria for the determined diagnoses.

2.1 Configuration Task

The following (simplified) assortment of digital cameras (see Table 1) and a set of inconsistent user requirements (see Table 2) for selecting a digital camera from the camera product table will serve as a working example to demonstrate how our approach works.

Our working example is represented as a configuration task in Table 2 (on the basis of Definition 1). As shown in Table 2, our example configuration consists of a variable set (V) with 10 variables (which are also listed in the first column of Table 1) and only one knowledge base constraint (c_1) which represents the set of available

Table 1: Digital Camera Product Table

	$Camera_1$	$Camera_2$	$Camera_3$	$Camera_4$	$Camera_5$
effectiveResolution	20.9	6.1	6.1	6.2	6.2
display	3.5	2.5	2.2	1.8	1.8
touch	yes	yes	no	no	no
wifi	yes	yes	no	no	no
nfc	no	no	no	no	no
gps	yes	yes	no	no	yes
videoResolution	UHD	UHD	No	UHD	4K
zoom	3.0	3.0	7.8	5.8	3.0
weight	475	475	700	860	560
price	659	659	189	2329	469

Table 2: An example of an inconsistent configuration task: CSP_{Lisa}

V, D	v_1: **effectiveResolution**: $\{6.1Megapixel, 6.2Megapixel, 20.9Megapixel\}$, v_2: **display**: $\{1.8inches, 2.2inches, 2.5inches, 3.5inches\}$, v_3: **touch**:$\{no, yes\}$, v_4: **wifi**:$\{no, yes\}$, v_5: **nfc**:$\{no, yes\}$, v_6: **gps**:$\{no, yes\}$, v_7: **videoResolution**: $\{No, UHD, 4K\}$, v_8: **zoom**:$\{3.0x, 5.8x, 7.8x\}$, v_9: **weight**: $\{475g, 560g, 700g, 860g, 1405g\}$, v_{10}: **price**: {€189, €469, €659, €2329, €5219}
C_{KB}	c_1: ($Camera_1 \vee Camera_2 \vee Camera_3 \vee Camera_4 \vee Camera_5$)
REQ_{Lisa}	c_2: **effectiveResolution**=20.9Megapixel c_3: **display**=2.5inches c_5: **wifi**=yes c_7: **gps**=yes c_9: **zoom**=5.8x

cameras shown in Table 1). User related preferences are defined in the set of user requirements (REQ_{Lisa}).

Definition 1 (Configuration Task). A configuration task can be defined as a $CSP(V,D,C)$. $V = \{v_1, v_2, \ldots, v_n\}$ represents a set of finite domain variables. $D = \{dom(v_1), dom(v_2), \ldots, dom(v_n)\}$ represents a set of variable domains where $dom(v_i)$ represents the domain of variable v_i. $C = C_{\text{KB}} \cup REQ$ where $C_{\text{KB}} = \{c_1, c_2, \ldots, c_q\}$ is a set of domain specific constraints (the configuration knowledge base) that restricts the possible combinations of variable values. $REQ = \{c_{q+1}, c_{q+2}, \ldots, c_t\}$ is a set of user requirements, which is also represented as constraints. A configuration (S) for a configuration task is a set of assignments $S = \{v_1 = a_1, v_2 = a_2, \ldots, v_n = a_n\}$ where $a_i \in$ dom(v_i) which is consistent with the constraints in C.

2.2 Diagnosis Task

In an interactive configuration scenario (see Table 1), a configuration (solution) may not be found due to the fact that some constraints are inconsistent (see Table 2). Such a "no solution could be found" dilemma is caused by at least one *conflict* between constraints in the knowledge base C_{KB} and the user requirements REQ (assuming the knowledge base C_{KB} to be consistent). Note that it could also be the case that C_{KB} itself is inconsistent [1]. Definition 2 introduces a formal concept of a conflict.

Definition 2 (Conflict). A conflict is a set of constraints $CS \subseteq C_{\text{KB}} \cup REQ$ which is inconsistent.

If we have an inconsistency in our knowledge base, we can say that $C_{\text{KB}} \cup REQ$ is a conflict set. Definition 3 introduces the concept of *minimal conflicts*.

Definition 3 (Minimal Conflict). A minimal conflict CS is a conflict (see Definition 2) where CS only contains constraints which are responsible for the conflict, s.t. $\nexists c_i \in CS : CS - \{c_i\}$ is inconsistent.

Our example contains three minimal conflict sets. $CS_1 = \{c_2, c_3\}$ because there is no product with c_2: **effectiveResolution**=$20.9 Megapixel$ and c_3: **display**=$2.5 inches$ in the product table C_{KB}, $CS_2 = \{c_2, c_9\}$ since there is no product with c_2: **effectiveResolution**=$20.9 Megapixel$ and c_9: **zoom**=$5.8x$, and $CS_3 = \{c_3, c_9\}$ there is no product with c_3: **display**=$2.5 inches$ and c_9: **zoom**=$5.8x$.

In such cases, we can help users to resolve the conflicts with a diagnosis Δ (a set of constraints). Assuming that C_{KB} is consistent, we can say that the knowledge base always will be consistent if we remove REQ. Removing Δ from REQ leads to a consistent knowledge base (see Definition 4).

Definition 4 (REQ Diagnosis Task). A user requirements diagnosis task is defined as a tuple (C_{KB}, REQ) where REQ is a set of given user requirements and C_{KB} represents the constraints part of the configuration knowledge base. A diagnosis for a REQ diagnosis task (C_{KB}, REQ) is a set $\Delta \subseteq REQ$, s.t. $C_{\text{KB}} \cup (REQ - \Delta)$ is consistent (which means that there is at least one solution). Δ is minimal if there does not exist a diagnosis $\Delta' \subset \Delta$, s.t. $C_{\text{KB}} \cup (REQ - \Delta')$ is consistent.

The feasibility (consistency) of the user requirements can be checked using a CSP solver [3,7]. If a CSP is consistent, there exists a solution i.e., $solve(CSP) \neq \emptyset$ where $solve(CSP : \{V, D, C\})$ includes a set of assignments S (a configuration/solution).

In Definition 5, we introduce the term *minimal diagnosis* which helps to reduce the number of constraints within a diagnosis.

Definition 5 (Minimal Diagnosis). A minimal diagnosis Δ is a diagnosis (see Definition 4) where there doesn't exist a subset $\Delta' \subset \Delta$ which has the diagnosis property.

The *REQ diagnosis task* triggered by the requirements of the user *Lisa* (REQ_{Lisa}) has three corresponding minimal diagnoses. The removal of the set $\Delta_{\text{Lisa}_1} = \{c_2, c_9\}$ or $\Delta_{\text{Lisa}_2} = \{c_3, c_9\}$ leads to a consistent configuration, i.e., $REQ_{\text{Lisa}} - \Delta_{\text{Lisa}_1} \cup C_{\text{KB}}$ is consistent and $REQ_{\text{Lisa}} - \Delta_{\text{Lisa}_2} \cup C_{\text{KB}}$ is consistent. This also holds for $\Delta_{\text{Lisa}_3} = \{c_2, c_3\}$.

2.3 Direct Diagnosis

Algorithmic approaches to provide efficient solutions for diagnosis problems are many-fold. Basically, there are two types of approaches which are *conflict-directed diagnosis* [13,18] and *direct diagnosis* [4].

Conflict-directed diagnosis algorithms calculate conflicts which are then used to find diagnoses. Their runtime performance is often not sufficient especially in real-time scenarios. *Direct diagnosis* algorithms determine diagnoses by executing a series of queries. These queries check the consistency of a constraint set without the need of pre-calculated conflict sets.

FLEXDIAG [4] is a direct diagnosis algorithm which determines one diagnosis at a time which indicates variable assignments of the original configuration that have to be changed such that a reconfiguration conform to the new requirements is possible. FLEXDIAG is *constraint-ordering sensitive*, i.e., depending on the ordering of the input constraints, different diagnoses (if more than one exists) could be determined.

2.4 Constraint Ordering

Quality and runtime performance of direct diagnosis algorithms are based on the ordering of the constraints in the set of user requirements: *the lower the importance of a constraint, the lower the index of the constraint*. The lower the index of a conflicting constraint, the higher the probability that this constraint is part of a diagnosis [5].

Users typically prefer to keep the important requirements and to change or delete (if needed) the less important ones [6]. The major goal of (model-based) diagnosis tasks is to identify the *preferred (leading) diagnoses* [8]. For the characterization of a preferred diagnosis, we will rely on a total ordering of the given set of constraints in *REQ*. Such a total ordering can be achieved, for example, by directly asking the customer regarding his/her preferences, by applying multi attribute utility theory where the determined interest dimensions are related to the attributes of *REQ*, or by applying the orderings determined by conjoint analysis [15].

2.5 Evaluation Criteria

We can evaluate the performance of a direct diagnosis algorithm, for example, based on *runtime performance*, *diagnosis quality* (in terms of minimality), and *combined performance* (runtime and minimality).

Runtime. $runtime(\Delta)$ represents the time needed by the diagnostic search to find Δ. This spent time can be measured in milliseconds or in terms of number of consistency checks (**#CC**) applied till a diagnosis is found. For a more accurate runtime measurement (excluding the operating system's effects on runtime, etc.), the number of consistency checks can be used.

Minimality. Diagnosis quality can be measured in terms of the degree of minimality of the constraints in a diagnosis, the cardinality of Δ compared to the cardinality of $\Delta_{\min}$. $|\Delta_{\min}|$ represents the cardinality of a corresponding minimal diagnosis. The highest (best) minimality can be 1 according to Formula 1.

$$minimality(\Delta) = \frac{|\Delta_{\min}|}{|\Delta|} \quad (1)$$

Combined. Since it is important to satisfy both evaluation criteria, runtime performance and minimality at the same time, we also evaluate the combined performance (see Formula 2). In this context, $combined(\Delta)$ increases if $minimality(\Delta)$ increases and/or $runtime(\Delta)$ decreases. This means, the direct diagnosis algorithm provides a diagnosis with a high minimality and low runtime while the combined performance is high.

$$combined(\Delta) = \frac{minimality(\Delta)}{runtime(\Delta)} \quad (2)$$

In order to improve the runtime performance of diagnostic search, FLEXDIAG [4] uses a parameter (*m*) that helps to systematically reduce the number of consistency checks but at the same time the minimality of diagnoses is deteriorated. In FLEXDIAG, the parameter *m* is used to control diagnosis quality in terms of minimality, accuracy, and the performance of diagnostic search. The higher the value of *m*, the higher the performance of FLEXDIAG and the lower the degree of diagnosis quality.

3 Related Work

The most widely known algorithm for the identification of minimal diagnoses is *hitting set directed acyclic graph* (HSDAG) [13]. HSDAG is based on conflict-directed hitting set determination and determines diagnoses based on breadth-first search. It computes minimal diagnoses (also minimal cardinality diagnoses) using minimal conflict sets which can be calculated, for example, by QUICKXPLAIN [6]. The major disadvantage of applying this approach is the need of predetermining minimal conflicts which can deteriorate diagnostic search performance [5]. Many different approaches to provide efficient solutions for diagnosis problems are proposed. One approach [23] focuses on improvements of HSDAG. Another approach [22] uses predetermined sets of conflicts based on binary decision diagrams. In diagnosis scenarios where the number of minimal diagnoses and their cardinality is high, the determination of diagnoses with standard conflict-based approaches becomes inefficient.

The direct diagnosis algorithm FLEXDIAG [4] utilizes an inverse version of QUICKXPLAIN [6] which finds a diagnosis directly from an inconsistent constraint set. FLEXDIAG is an extension of FASTDIAG [5] which assures diagnosis determination within certain time limits by systematically reducing the number of needed solver calls. The authors claim that this specific interpretation of anytime diagnosis leads to *a trade-off between diagnosis quality (evaluated, e.g., in terms of minimality) and the time* needed for diagnosis determination. Our proposed constraint ordering approach *LCO* improves their direct diagnosis approach [4] in terms of diagnosis quality and at the same time in terms of runtime performance.

The approach of [17] determines diagnoses directly. The authors reduce the number of consistency checks by avoiding the computation of conflict sets. Their approach is similar to [5], but introduces a new pruning rule. They compared their approach with the standard technique (based on QUICKXPLAIN [6] and HSDAG

[13]). In their experiments, they show that direct diagnosis outperforms the standard diagnosis approach in terms of runtime. In their approach, authors collect the constraint ordering directly from users (interactively). Compared to the work presented in [17], our approach learns the constraint ordering from historical inconsistent user requirements and their preferred diagnoses.

The relevance of constraint ordering in direct diagnosis scenarios has already been discussed [4, 17]. Based on historical transactions (user interaction data), we predict the constraints of high relevance for a user based on *matrix factorization* which is a model-based collaborative filtering approach [9]. We learn a constraint ordering where the predicted most important constraints have the highest ordering. This is due to the fact that the implemented direct diagnosis algorithm FLEXDIAG first starts to search a diagnosis in the lowest-ranked constraints. Consequently, constraints with a higher ranking have a lower probability of being part of a diagnosis.

4 Learned Constraint Ordering (LCO) for Direct Diagnosis

In this paper, our goal is to solve *the quality-runtime performance trade off problem* of direct diagnosis. For this purpose, our proposed method learns constraint ordering heuristics based on historical transactions in an offline phase and then, in an online phase, it employs a direct diagnosis algorithm on the re-ordered constraints. In this paper, we demonstrate and evaluate our approach based on the direct diagnosis algorithm FLEXDIAG [4].

Our contributions in this context are the following. We utilize the *constraint-ordering sensitivity* of direct diagnosis and propose a novel learning approach for constraint ordering.

In order to increase diagnosis quality and runtime performance of FLEXDIAG at the same time, we propose matrix factorization based constraint ordering (*LCO*) for the diagnosis of inconsistent constraints (see Algorithm 1). In our approach, we build a *sparse matrix (R)* by exploiting *historical purchase transactions with inconsistent user requirements and preferred minimal diagnoses* and a new inconsistent set of user requirements (constraints). The estimated dense matrix (as a result of matrix factorization of the sparse matrix) provides the input in terms of importance estimates for determining a new diagnosis. According to the importance estimates determined by matrix factorization, constraints are reordered in before employing direct diagnosis.

Data: Historical transactions and an inconsistent *CSP*
Result: Minimal diagnosis for the inconsistent *CSP*
if *offline phase not yet executed* **then**
    sm= build_sparse_matrix (historical transactions);
    dm= factorize_and_get_dense_matrix(*sm*);
    LCO= extract_constraint_orderings_from_the_dense_matrix (*dm*);
else
    T_1= find_the_closest_transaction_using_Euclidean_Distance(*CSP*);
    LCO_1= get_constraint_ordering_for(T_1);
    CSP_1= reorder_constraints(*CSP*,LCO_1);
    Diagnosis= DirectDiagnosisAlgorithm(CSP_1);
    return *Diagnosis*;
end

Algorithm 1: Pseudocode of Learned Constraint Ordering (LCO)

In the following, we explain our approach by demonstrating how it works with the example diagnosis task ($REQ_{\text{Lisa}} \cup C_{\text{KB}}$). In this context, we show the experimental evaluations of our approach on the basis of real-world configuration knowledge bases. In our camera configuration example, *Lisa* provides her requirements which are inconsistent with the camera product table, i.e., no solution could be found for *Lisa's* requirements. Therefore, her requirements need to be diagnosed. Following our approach, we first calculate a new constraint ordering (*LCO*) and then solve the diagnosis task ($REQ_{\text{Lisa}} \cup C_{\text{KB}}$) using the identified (new) constraint ordering.

4.1 Offline Phase: Learning from Historical Transactions

Our proposed method needs an offline phase in which various constraint ordering heuristics are learned based on historical transactions. For offline learning, matrix factorization techniques and *historical transactions with inconsistent user requirements* are employed (see Table 3). An example of using a learned heuristics is a solution search starting with the variable "resolution" and the corresponding value "20.9" and continuing with the other values of "resolution" in the learned order. After the assignment of the variable "resolution" is satisfied, continue with the next variable with its learned value orderings until all variable values are consistent with the given set of constraints.

In Table 3, for each user, we have an inconsistent set of user requirements (which leads to "no solution"). After a no-solution situation, some of the users (in our case, Alice, Tom, and Joe in Table 3) decided to buy a product (*Purchase*) which does not completely satisfy their requirements. Consequently, they had to change their initial requirements which is presented as Δ_{min}. These historical transactions which are completed with a purchase are *complete historical transactions*. The rest of historical transactions, in which users did not complete their transactions with a purchase (e.g. Bob and Ray in Table 3), is called *incomplete historical transactions*. In this context, we estimate diagnoses of *incomplete historical transactions* using matrix factorization and we take into account only historical transactions with minimal diagnoses.

Table 3: Historical transactions with inconsistent user requirements.

	Alice	Bob	Tom	Ray	Joe
c_2: resolution	-	6.1	20.9	20.9	6.2
c_3: display	3.5	2.2	-	2.5	2.2
c_4: touch screen	-	-	yes	yes	-
c_5: wifi	-	-	yes	yes	-
c_6: nfc	-	yes	-	-	yes
c_7: gps	-	yes	yes	yes	no
c_8: video resolution	UHD	-	UHD	UHD	UHD
c_9: zoom	3.0	5.8	3.0	5.8	7.8
c_{10}: weight	560	700	475	475	-
c_{11}: price	469	189	469	-	189
Purchase	Camera_1	-	Camera_1	-	Camera_3
Δ_{min}	c_{10}, c_{11}	-	c_3, c_{11}	-	c_2, c_6, cv_7

Based on the historical transactions in Table 3, we know that, the product $Camera_1$ is purchased by *Alice*. This means, Alice has changed her requirements c_{10}: **weight**=$560g$ and c_{11}: **price**=469 and purchased the product $Camera_1$ which has **weight**=$475g$ and **price**=659. Therefore, Δ_{Alice}=$\{c_{10}, c_{11}\}$ is a diagnosis for the requirements diagnosis task ($REQ_{\text{Alice}} \cup C_{\text{KB}}$). When we eliminate the diagnosis constraints from the inconsistent requirement set, the diagnosed requirement set becomes REQ'_{Alice} = $\{$**effective Resolution**=20.9 Megapixel, **display**=$3.5 inches$, **touch**=*yes*, **wifi**=*yes*, **nfc** =*no*, **gps**=*yes*, **videoResolution**=$UHD - U4K/3840x2160$, **zoom**=$3.0x\}$. Based on REQ'_{Alice}, the found solution set is $\{Camera_1\}$ which includes the products purchased by Alice ($Camera_1$).

4.1.1 The Sparse Matrix

Matrix factorization based collaborative filtering algorithms [10] introduce a rating matrix R (a.k.a., user-item matrix) which describes preferences of users for the individual items the users have rated. Thereby, R represents an $m \times n$ matrix, where m denotes the number of users and n the number of items. The respective element $r_{u,i}$ of the matrix R describes the rating of the item i made by user u. Given the complete set of user ratings, the recommendation task is to predict how the users *would* rate the items which have not yet been rated by these users.

In our approach, we build a sparse matrix R (user-constraint matrix) using inconsistent historical transactions as shown in Table 4-(a) where columns represent constraints. Therefore, each row of the sparse matrix R represents a set of user requirements (the left half) and their corresponding diagnoses (the right half) if available. User requirements are presented in their normalized values in the range of 0-1, and diagnoses are presented with the presence (1)/non-presence (0) of user requirements.

If there are non-numeric domains in the problem, they are enumerated. For example, the domain v_7: **videoResolution**: $\{No, UHD, 4K\}$ is enumerated as v_7: $\{0, 1, 2\}$. Besides, domain ranges of all constraints in REQ are mapped to [0..1] since matrix factorization needs to use the same range for all values in the matrix. For this purpose, we have employed *Min-Max Normalization* [21] according to Formula 3.

$$v_{\text{i\_norm}} = \frac{v_{\text{i}} - dom(v_{\text{i}})_{\text{min}}}{dom(v_{\text{i}})_{\text{max}} - dom(v_{\text{i}})_{\text{min}}} \tag{3}$$

In our camera configuration example, in the left half of the first row of *R*, we set the user requirements of *Alice*. As shown in Table 3, she prefers 3.5 inches displays, i.e. c_3: **display**=3.5*inches*. In Table 4, the assigned value of **display** is normalized using Formula 3 and represented as 1. In the right half of the first row of *R*, we set the diagnoses probabilities. If a constraint c_{i} exists in Δ_{user}, it's corresponding diagnosis probability δ_{i} is 1, otherwise to 0. We know that Δ_{Alice}=$\{c_{10}, c_{11}\}$ (see Table 3), consequently we set δ_{10} and δ_{11} to 1 and the rest to 0.

4.1.2 Matrix Factorization

In terms of *matrix factorization* [11, 12], the sparse matrix *R* is decomposed into an $m \times k$ *user-feature matrix P* and a $k \times n$ *constraint-feature matrix* Q^T which both are used to find the estimated dense matrix PQ^{T}. In this context, *k* is a variable parameter which needs to be adapted to optimize the prediction quality for the test data.

In our example, we apply matrix factorization to the sparse matrix in Table 4-(a). Then, the estimated matrix is obtained as shown in Table 4-(b) which includes the estimated diagnoses for *Bob* and *Ray*.

Table 4: Matrix factorization estimates a dense matrix PQ^{T} (b) which closely approximates the sparse matrix *R* (a).

(a) The sparse matrix (*R*)

	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	p_{c2}	p_{c3}	p_{c4}	p_{c5}	p_{c6}	p_{c7}	p_{c8}	p_{c9}	p_{c10}	p_{c11}
Alice		1					0.5	0	0.09	0.05	0	0	0	0	0	0	0	0	1	1
Bob	0	0.23			1	1		0.58	0.24	0										
Tom	1		1	1		1	0.5	0	0	0.05	0	1	0	0	0	0	0	0	0	1
Ray	1	0.41	1	1		1	0.5	0.58	0											
Joe	0.006	0.23			1	0	0.5	1		0	1	0	0	0	1	1	0	0	0	0

(b) The estimated dense matrix (PQ^{T})

	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	p_{c2}	p_{c3}	p_{c4}	p_{c5}	p_{c6}	p_{c7}	p_{c8}	p_{c9}	p_{c10}	p_{c11}
Alice	1.4	0.2	1.3	1.5	0.3	1.2	0.9	0.6	-0.2	-0.3	0.4	0.2	-0.4	-0.4	0.4	0.4	-0.4	0.2	0.4	1.1
Bob	0.9	0.3	1.2	1.3	1.1	1.3	0.9	1	0	-0.3	0.7	0	-0.4	-0.4	1.1	1.1	-0.4	0.6	0.1	0.5
Tom	1.5	0.3	1.5	1.6	0.3	1.4	0.9	0.6	-0.2	-0.3	0.4	0.3	-0.4	-0.4	0.4	0.4	-0.4	0.3	0.4	1.1
Ray	1.4	0.3	1.5	1.6	0.7	1.5	0.9	0.9	-0.1	-0.3	0.8	0.1	-0.4	-0.4	0.6	0.6	-0.4	0.7	0.2	0.6
Joe	0.7	0.2	0.9	1.1	1.2	0.9	0.8	1.1	0.1	-0.3	0.8	-0.1	-0.3	-0.3	1.1	1.1	-0.4	0.4	0.1	0.4

4.2 Online Phase: Diagnosing Active Transactions

After calculating the estimated matrix in the offline phase, in the online phase we diagnose active transactions which include inconsistencies as in our working example. In active transactions, users still did not leave the configuration session and need real-time help to remove inconsistencies in their requirements to decide on a product to purchase. Therefore, the configuration system should provide a relevant diagnosis in a reasonable time (before users leave the system without a purchase).

4.2.1 The Most Similar Historical Transaction

Using Formula 4, we find the historical transaction which is the most similar one to the new set of inconsistent requirements. In this context, HT represents a historical transaction, AT represents the active transaction, $HT.c_i$ represents the value of each requirement in the estimated dense matrix PQ^T, and $AT.c_i$ represents the value of each requirement in the active transaction. i represents a constraint index value in the REQ of AT.

$$min(\sqrt{\sum_{i \in AT.REQ} \|HT.c_i - AT.c_i\|^2}) \tag{4}$$

In our camera configuration example, the most similar historical transaction to the active transaction of *Lisa* is the transaction of *Ray*. Therefore, to diagnose REQ_{Lisa}, we use LCO_{Ray}: $\{c_2, c_9, c_6, c_7, c_{11}, c_{10}, c_3, c_4, c_5, c_8\}$. When we only consider user requirements of *Lisa*: $\{c_2, c_3, c_5, c_7, c_9\}$, we obtain the constraint ordering for Lisa LCO_{Lisa}: $\{c_2, c_9, c_7, c_3, c_5\}$.

4.2.2 Direct Diagnosis with LCO

After the most similar historical transaction has been found and its constraint ordering has been applied to the active transaction's user requirements, the direct diagnosis algorithm is employed. We employ FLEXDIAG as the direct diagnosis algorithm with its parameter *m* which is used to control diagnosis quality.

In our working example, user constraints of the diagnosis task are reordered using LCO_{Lisa} as $\{c_2, c_9, c_7, c_3, c_5\}$ and a minimal diagnosis $\Delta = \{c_2, c_9\}$ is found by FLEXDIAG(m=1) with performance results (on the basis of evaluation criteria given in Section 2.5): #CC = 4, minimality = 1, and combined = 0.250. However, when we employ the diagnosis algorithm FLEXDIAG(m=1) on the default order of user constraints $\{c_2, c_3, c_5, c_7, c_9\}$, the same diagnosis $\Delta = \{c_2, c_9\}$ is found with performance results: #CC = 8, minimality = 1, and combined = 0.125. Therefore, using *LCO* with FLEXDIAG(m=1), we improve the performance of diagnosis when diagnosing the working example.

In the following, we discuss the effects of our approach on a direct diagnosis algorithm FLEXDIAG, based on the evaluation criteria: *runtime* (based on the number of consistency checks), *minimality*, and *combined*. The search trees of FLEXDIAG with *LCO* (Tree-2 and Tree-4) have better combined performance compared to the search trees of FLEXDIAG without *LCO* (Tree-1 and Tree-3). When *m*=1, *LCO* improved the combined performance (see Formula 2) with the ratio 32% (0.166 instead of 0.125). If *m*=2, *LCO* improves the combined performance with the ratio 100% (0.250 instead of 0.125) and the minimality with the ratio 50% whereas the runtime is not improved. Consequently, as also observed in the example search trees, *LCO* improves runtime performance and minimality of diagnosing.

5 Experimental Evaluation

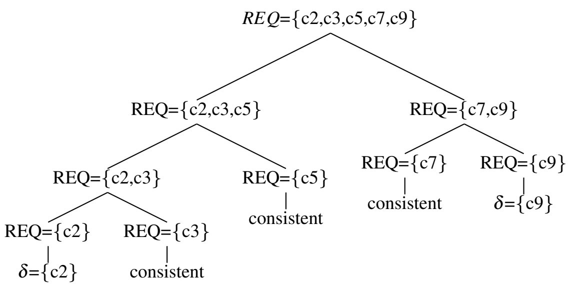
5.1 Settings

We have implemented our approach in Java and tested on a computer with an Intel Core i5-5200U, 2.20 GHz processor, 8 GB RAM and 64 bit Windows 7 Operating System and Java Run-time Environment 1.8.0. Constraint satisfaction problems have been solved by *Choco*[1] which is a Java library for constraint satisfaction problems with a FlatZinc (the target language of MiniZinc) parser. For matrix factorization, we have used the *SVDRecommender* of Apache Mahout [16].[2]

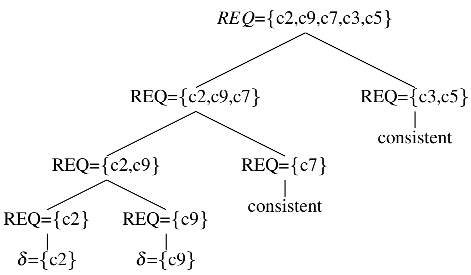
5.2 Datasets

We have used *Minizinc-2016* benchmark problems [19] where each problem includes five data files with file extension ".*dzn*".[3]

Tree-1: FLEXDIAG (M=1): $\Delta = \{c2, c9\}$
#CC = 8, **minimality = 1**, and **combined = 0.125**



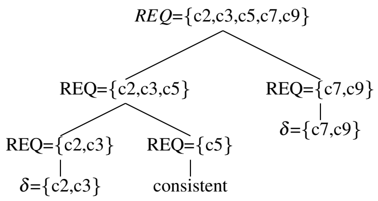
TREE-2: FLEXDIAG (M=1) with LCO: $\Delta = \{c2, c9\}$
#CC = 6, **minimality = 1**, and **combined = 0.166**



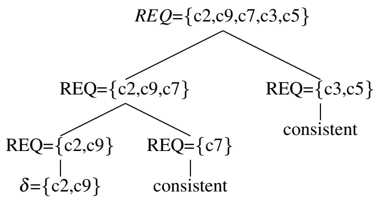
TREE-3: FLEXDIAG (M=2): $\Delta = \{c2, c3, c7, c9\}$
#CC = 4, **minimality = 0.5**, and **combined = 0.125**

---

[1] http://www.choco-solver.org/
[2] using the latent factor *k*=100 and the number of iterations = 1000
[3] http://www.minizinc.org/challenge2016/results2016.html



TREE-4: FLEXDIAG (M=2) with LCO: $\Delta = \{c2, c9\}$
#CC = 4, **minimality = 1**, and **combined = 0.250**:



In order to obtain historical and active transactions based on these benchmark problems, we randomly generated 5000 sets of inconsistent user requirements (each with *N* constraints) based on integer variables. We have employed the java-code snippet below which inserts 10 additional constraints to each benchmark problem and always leads an inconsistency:

```
//PUSH INCONSISTENT CONSTRAINTS INTO the CSP (solver)
Constraint []c = new Constraint[N];
int reqs = new int [numberOfVariables];
while(true){
for (int i=0;i<N;i++){
int index= 0;
do{index = (int)(Math.random() * numberOfVariables);}
while(reqs[index]!=-1);
int lb = var[i].getLB();int ub = var[i].getUB();
int v = ((int)(Math.random() * ub-lb))+lb;   reqs[index]=v;
c[k] =          IntConstraintFactory.arithm(intVars[index],"=",v);
solver.post(c[k]);} //INSERT GENERATED CONSTRAINTS
if(solver.findSolution()){ //IF CONSISTENT, TRY AGAIN
for (int k=0;k<N;k++) solver.unpost(c[k]);
Arrays.fill(reqs, -1);   } else break;}
```

5.3 Comparative Methods

We compare our approach directly with FLEXDIAG[4]. In this article, we do not compare *LCO* with more traditional diagnosis approaches – for related evaluations we refer the reader to [4] where detailed analyses of FLEXDIAG can be found. We used FLEXDIAG to show the minimality improvements of using *LCO* when $m \geq 2$. As baseline, we evaluated FLEXDIAG-without constraint ordering. *#REQ* represents the number of constraints in a set of inconsistent requirements.

To evaluate *LCO*, we analyzed the three aspects (1) runtime performance, (2) diagnosis quality (in terms of minimality, see Formula 1), and (3) combined performance (see Formula 2). We observed that our approach *LCO* outperforms the baseline versions (see Table 5).

5.4 Results

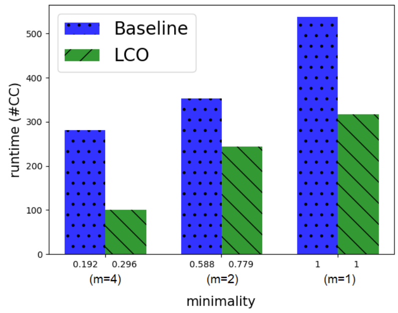
As discussed throughout this paper, our main objective is to improve the combined performance (runtime performance and diagnosis quality at the same time). We have compared our approach *LCO* with the baseline *no constraint ordering*. In both cases, for diagnostic search, FLEXDIAG is used with three different *m* values 1, 2, and 4.

As shown in Table 5, based on the average value (in the last row), *LCO* outperforms the baseline in terms of *runtime* and *minimality* since with each *m* value (1, 2, and 4), *LCO* has lower runtime than the baseline whereas its minimality is higher (or equal) compared to the baseline with each *m* value (1, 2, and 4).

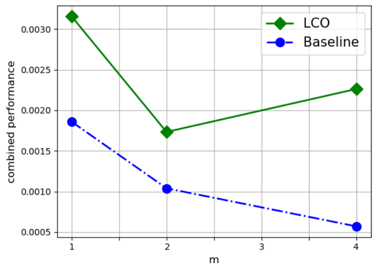
Based on the results in Table 5, we present two comparisons in Figure 1 where relations between performance indicators and the number of constraints in the set of user requirements are shown.

In Figure 1-(a), we observe that *minimality* improves (or the parameter *m* decreases) when *runtime* increases. As observed, the number of consistency checks (#CC) are at each *m* (*m*=1, 2, and 4) lower when *LCO* is used. Moreover, at each *m* (*m*=1, 2, and 4), *LCO* also provides better minimality results.

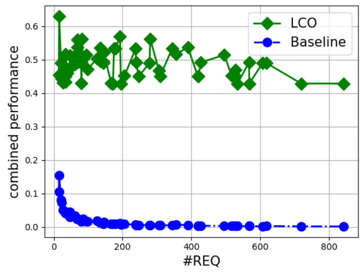
The combined performance results are shown in Figure 1-(b) and Figure 1-(c). Deviations in the results of *LCO* are more visible than the baseline, because *LCO* has greater performance values. When we zoom into the results of the baseline, we also observe similar deviations due to the variations in the problems. As observed, our approach improves the combined performance significantly.



(a) Runtime (#CC)



(b) Combined vs *m*



(c) Combined vs *#REQ*

Fig. 1: Comparison Graphs based on the Experimental Results in Table 5

6 Conclusions

In this paper, we proposed a novel learning approach for constraint ordering heuristics to solve *the quality-runtime performance trade off problem* of direct diagnosis. We employed matrix factorization for learning based on historical transactions. Taking the advantage of learning from historical transactions, we calculated possible constraint ordering heuristics in an offline phase for solving new diagnosis tasks in online phase.

In particular, we applied our constraint ordering heuristics to reorder user constraints of diagnosis tasks and employed a direct diagnosis algorithm FLEXDIAG [4] to diagnose the reordered constraints of the diagnosis tasks. The reason to choose this diagnosis algorithm is that *the quality-runtime performance trade off problem* is much more obvious in FLEXDIAG when its *m* parameter is increased. However, our approach can be also applicable to other direct diagnosis approaches (e.g. [17]). We compared our approach with a baseline: FLEXDIAG without heuristics. According to our experimental results, our approach (*LCO*) solves the *the quality-runtime perfor-*

Table 5: Experimental results based on Minizinc-2016 Benchmark problems.

EXPERIMENTAL RESULTS																	
Minizinc 2016 Benchmark			Inconsistencies		BASELINE						OUR APPROACH						
					runtime			*minimality*			*runtime*			*minimality*			
.mzn	*.dzn*	#vars	#REQs	$\lvert\Delta_{\min}\rvert$	m=1	m=2	m=4	m=1	m=2	m=4	m=1	m=2	m=4	m=1	m=2	m=4	
1. cc_base	test.02	136	68	5	27	23	15	1	0.333	0.25	19	15	6	1	0.5	0.333	
	test.06	478	239	23	120	80	60	1	0.333	0.166	60	53	22	1	1	0.333	
	test.11	159	80	5	32	27	18	1	0.5	0.25	20	16	7	1	1	0.333	
	test.13	291	146	11	73	42	36	1	0.5	0.166	36	32	13	1	1	0.333	
	test.20	688	344	28	172	98	86	1	0.333	0.166	86	76	29	1	1	0.333	
2. celar	CEL.6-S0	557	279	21	139	80	70	1	1	0.143	70	62	23	1	1	0.333	
	CEL.6-S4	1136	568	45	284	189	126	1	0.333	0.25	162	114	52	1	1	0.25	
	CEL.7-S4	1137	569	45	227	190	126	1	0.333	0.143	162	126	52	1	0.5	0.333	
	graph05	2680	1340	99	536	447	335	1	0.333	0.143	383	268	122	1	1	0.25	
	scen07	6331	3166	211	1583	1055	703	1	1	0.166	791	703	288	1	1	0.25	
3. step1	kb128_11	17390	8695	790	4348	2484	1932	1	0.5	0.25	2174	1739	725	1	1	0.25	
	kb128_14	17390	8695	828	4348	2898	2174	1	0.333	0.166	2174	1932	725	1	0.5	0.333	
	kb128_16	17390	8695	870	4348	2484	2174	1	1	0.25	2174	1739	725	1	1	0.25	
	kb128_17	17390	8695	870	3478	2898	2174	1	0.333	0.166	2484	1739	725	1	0.5	0.25	
	kb192_10	47278	23639	1970	9456	6754	5253	1	0.5	0.166	6754	5253	2149	1	0.5	0.333	
4. depot	att48_6	77	39	3	19	11	9	1	1	0.166	10	9	3	1	1	0.25	
	rat99_5	69	35	3	14	12	9	1	0.333	0.143	9	8	3	1	1	0.25	
	rat99_6	77	39	4	19	11	9	1	0.333	0.25	10	8	4	1	1	0.25	
	st70_5	69	35	2	17	10	9	1	0.5	0.166	10	8	3	1	0.5	0.25	
	ulysses	69	35	2	17	10	9	1	0.333	0.166	10	7	3	1	1	0.333	
5. dcmst	c_v15_d7	494	247	18	124	71	62	1	0.5	0.166	62	49	22	1	0.5	0.25	
	c_v20_d5	856	428	39	214	143	107	1	0.333	0.143	122	95	39	1	0.5	0.25	
	s_v20_d4	390	195	14	78	65	49	1	1	0.25	49	39	18	1	1	0.333	
	s_v20_d5	385	193	14	77	64	48	1	0.333	0.166	48	43	16	1	0.5	0.333	
	s_v40_d5	1214	607	45	304	173	135	1	1	0.2	173	121	51	1	0.5	0.25	
6. filter	ar_1_3	121	61	6	30	17	13	1	0.333	0.25	17	12	6	1	1	0.25	
	dct_1_3	189	95	9	38	32	21	1	0.5	0.143	24	19	9	1	1	0.333	
	ewf_1_2	139	70	6	35	20	15	1	1	0.25	17	14	6	1	0.5	0.333	
	fir_1_3	92	46	4	18	15	10	1	1	0.166	13	10	4	1	1	0.333	
	fir_1_4	92	46	3	18	15	12	1	0.333	0.2	12	10	4	1	0.5	0.333	
7. gbac	UD3	1687	844	77	422	241	211	1	0.5	0.25	241	187	70	1	1	0.25	
	UD6	561	281	24	140	94	62	1	0.333	0.166	80	62	23	1	1	0.333	
	UD10	619	310	24	155	103	77	1	1	0.2	77	62	28	1	1	0.25	
	UD3	1938	969	81	388	277	242	1	0.5	0.143	242	194	88	1	1	0.25	
	UD5	1439	720	55	360	206	160	1	0.5	0.2	180	160	65	1	1	0.333	
8. gfd-sch.	n25f5.	344	172	11	69	57	43	1	0.333	0.166	49	34	14	1	0.5	0.333	
	n35f5.	477	239	20	95	80	53	1	0.333	0.25	60	48	20	1	0.5	0.25	
	n55f2.	612	306	21	122	102	77	1	1	0.166	87	61	28	1	0.5	0.333	
	n60f7.	1236	618	54	247	177	155	1	0.5	0.143	177	137	56	1	0.5	0.333	
	n180f.	4950	2475	177	1238	825	550	1	0.5	0.143	619	495	225	1	0.5	0.25	
9. map.	m2x2_1	197	99	9	49	33	22	1	0.333	0.25	25	22	8	1	0.5	0.333	
	m2x2	358	179	16	90	60	40	1	1	0.2	45	40	16	1	1	0.333	
	m3x3	838	419	38	210	120	105	1	0.333	0.166	105	93	35	1	0.5	0.333	
	m4x4	841	421	28	168	120	105	1	1	0.25	120	93	38	1	0.5	0.25	
	ring	411	206	15	82	59	46	1	0.5	0.2	51	46	19	1	1	0.25	
10. m.dag	25_01	136	68	7	34	19	15	1	0.333	0.166	19	15	6	1	0.5	0.333	
	25_03	160	80	5	32	27	18	1	0.333	0.143	20	18	7	1	1	0.25	
	25_04	133	67	6	27	19	17	1	0.333	0.166	17	15	6	1	1	0.333	
	25_06	164	82	8	33	27	18	1	0.5	0.2	23	16	7	1	1	0.333	
	31_02	169	85	6	42	24	19	1	0.5	0.2	21	17	8	1	1	0.25	
11. mrcp.	j30_1_10	712	356	28	178	102	89	1	1	0.143	89	79	32	1	0.5	0.333	
	j30_15_5	333	167	12	67	56	37	1	0.5	0.2	42	37	15	1	0.5	0.333	
	j30_17_10	992	496	34	198	142	124	1	1	0.25	142	99	41	1	0.5	0.25	
	j30_37_4	780	390	28	195	111	98	1	0.5	0.2	111	78	33	1	0.5	0.25	
	j30_53_3	350	175	12	70	58	44	1	0.333	0.25	44	39	15	1	1	0.25	
12. nfc	12_2_5	31	16	1	8	5	3	1	0.333	0.2	4	3	1	1	1	0.25	
	12_2_10	29	15	1	6	4	4	1	1	0.166	4	3	1	1	1	0.333	
	18_3_5	45	23	2	11	6	6	1	1	0.143	6	5	2	1	0.5	0.333	
	18_3_10	42	21	2	11	7	5	1	1	0.2	5	5	2	1	0.5	0.333	
	24_4_10	54	27	2	11	8	7	1	0.5	0.25	7	5	2	1	1	0.25	
13. oocsp	030_e6_cc	1034	517	43	207	148	129	1	1	0.25	129	103	43	1	0.5	0.25	
	030_ea4_cc	1068	534	40	267	178	119	1	0.333	0.25	153	119	45	1	0.5	0.333	
	030_f7_cc	1058	529	44	265	151	132	1	1	0.166	151	106	44	1	1	0.25	
	030_mii8	1053	527	48	263	176	117	1	1	0.25	132	117	44	1	0.5	0.333	
	100_r1	3433	1717	149	858	572	429	1	0.333	0.25	429	381	143	1	1	0.333	
14. pc	28-4-7-1	252	126	12	50	42	28	1	0.333	0.143	32	25	11	1	1	0.25	
	30-5-6-2	270	135	9	54	39	34	1	1	0.143	39	27	11	1	1	0.333	
	30-5-6-8	270	135	11	54	45	30	1	1	0.143	34	27	11	1	0.5	0.333	
	32-4-8-2	288	144	10	58	41	36	1	0.333	0.2	36	32	13	1	1	0.333	
	32-4-8-5	288	144	10	58	41	36	1	0.5	0.2	36	29	13	1	1	0.333	
average		2349	1174	101	530	357	276	1	0.588	0.192	315	249	102	1	0.779	0.296	

mance trade off problem by improving the diagnosis quality (in terms of minimality) and the runtime performance at the same time.

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